III. *On the expediency of assigning specific names to all such functions of simple elements as represent definite physical properties; with the suggestion of a new term in mechanics; illustrated by an investigation of the machine moved by recoil, and also by some observations on the Steam Engine. By* DAVIES GILBERT, *Esq. M. P. V. P. R. S. &c.*

Read January 25, 1827.

THE expediency of distinguishing by separate appellations, all such functions of simple elements as measure the intensity of physical properties, will be rendered obvious by referring to the well known controversy respecting motion.

Scarcely had the principles which regulate the action of bodies in motion become subjected to mathematical calculation, when a dispute arose as to the measure of motion itself; a dispute conducted with much more vehemence and acrimony than might be supposed incident to the nature of an abstract subject.

Several individuals of the greatest learning and reputation contended that weight, multiplied by velocity, ($w \times v$) gave a product always proportionate to the motion of bodies, as was proved by a comparison of their inertiæ, by all the properties relative to the common centre of gravity in planetary systems, &c. &c. &c.; while other persons, scarcely inferior to the former, adverting to the collision of elastic bodies, and to the extremely curious property of motion, the conservatio virium vivarum, contended with equal confidence, that the true measure of motion was weight multiplied into the velocity squared ($w \times v^2$); till, in the early part of the last century it was fortunately observed, that the different properties indicated by these two functions were not, in any respect, at variance with each other; and the terms *momentum* and *impetus* reconciled all opinions, by removing every ground for dispute.

For a full and detailed explanation of this subject, I would refer to the admirable treatise on "The Rectilinear Motion and Rotation of Bodies," by the late GEORGE ATWOOD, Fellow of Trinity College, Cambridge, F. R. S. printed in 1784; and to the Bakerian Lecture in the Philosophical Transactions for 1806, by an existing member of this Society, who has never touched any point of science that he has not elucidated and adorned.

In this Lecture it is observed, that neither impetus nor momentum have usually much to do with the action of ordinary machines; which is undoubtedly true; since neither of these functions measures directly their efficient power. The criterion of their efficiency is force multiplied by the space through which it acts ($f \times s$); and the effect which they produce, measured in the same way, has been denominated *duty*, a term first introduced by Mr. WATT, in ascertaining the comparative merit of steam engines, when he assumed one pound raised one foot high, for what has since been called, in other countries, the dynamic unit.* And by this criterion, one bushel of coal, weighing 84lbs. has been found to perform a duty of thirty, forty, and even fifty millions; augmenting

* The dynamic unit used in France is a cubic metre (3,2809 feet) of water raised through a metre in height = 7220 pounds of water one foot high, being to a million as 1 : 138,5 as 72 : 10000 nearly; the log. 2,1413966.

with improvements chiefly in the fire place, which produce a more rapid combustion, with consequently increased temperature, and a more complete absorption of the generated heat; in addition to expansive working, and to the use of steam raised considerably above atmospheric pressure.

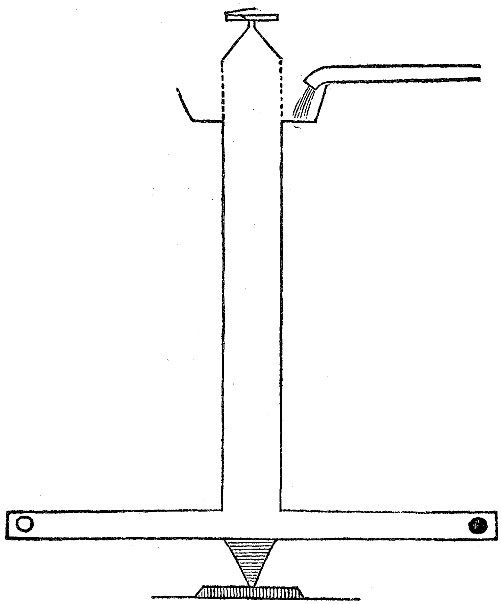
Since therefore a machine is efficient in producing duty, or effect, in proportion to the force applied, multiplied into the space through which it acts, I propose to denominate this function $(f \times s)$ *efficiency;* retaining the word *duty* for a similar function indicative of the work performed.

And it is obvious, that by a comparison of these two quantities, the *efficiency* expended on any machine, and the *duty* performed by it, an exact measure will be ascertained of its intrinsic worth.

Having never seen a full investigation of the engine moved by recoil, I am partly, on that account, induced to take it as an example of the utility to be derived from the use of this new term; and I am in part also induced to do so, by the circumstance of a most respectable society having last year held out the properties of BARKER's mill to the public, as fit objects for inquiry.

To investigate the Recoil engine.

Let every thing be taken in the abstract, and after a manner most advantageous to the machine. Suppose friction to vanish, and that the perpendicular tube, and the arms, are so capacious as to allow of the fluid issuing with a velocity equal to what a body would acquire by falling in free space, through an elevation equal to that of the perpendicular tube, or through, what is usually termed, the head. See Principia, B. 2. § 7.



Let $l =$ the space in English feet through which a body freely descends in one second of time.

Then $2\,l =$ the velocity acquired.

$h =$ the length of the perpendicular tube, or the actual height of the head.

Then $2\sqrt{hl} =$ the velocity due to the head.

Let $R =$ the length of either arm from the centre to the orifice.

$x =$ the velocity with which that point moves in revolving, expressed by feet in a second.

Then $\frac{x^2}{2\,l\,R} =$ the centrifugal force at the orifice.

And since the force exerted by each particle of the fluid varies as its distance from the centre

$\frac{x^2}{2\,l\,R} \times \frac{R}{2} = \frac{x^2}{4\,l} =$ the whole pressure from centrifugal force. An expression independent of the length of the arm; and the slightest reflection will make it appear that no alteration can arise from deflecting the form of the arm out of a straight line.

Then, since the pressure exerted by the actual head is expressed by its height (h), and the pressure from centrifugal force by $\frac{x^2}{4l}$,

$h + \frac{x^2}{4l}$ = the pressure sustained, or the virtual head, and $\left(h + \frac{x^2}{4l}\right) \times x$ = the efficiency applied to the machine, in relation however, not to the expenditure of fluid corresponding to the height (h), but to an expenditure greater in the proportion of $\sqrt{\left(1 + \frac{x^2}{4hl}\right)}$ to 1. But a portion of this efficiency must, of necessity, be expended in giving the rotary velocity (x) to the quantity of fluid issuing $= \sqrt{1 + \frac{x^2}{4lh}}$. Now the height due to x is $\frac{x^2}{4l}$, consequently $\sqrt{1 + \frac{x^2}{4lh}} \times \frac{x^2}{4l} \times x$ = the expenditure of efficiency in producing the rotary motion. And $\left(h + \frac{x^2}{4l} - \sqrt{1 + \frac{x^2}{4hl}} \times \frac{x^2}{4l}\right) \times x$ = the efficiency applied to the machine capable of producing duty.

Now substitute for x, (the velocity due to h) ($2\sqrt{hl}$) multiplied by an arbitrary quantity y; then $\sqrt{1 + \frac{x^2}{4hl}}$ (or the quantity of fluid issuing) $= \sqrt{1 + y^2}$ and $(y + y^3 - \sqrt{1 + y^2} \times y^3) \times 2 h^{\frac{3}{2}} l^{\frac{1}{2}}$ = the efficiency; and dividing by $\sqrt{1 + y^2}$ $\sqrt{1 + y^2} \times y^3 \times 2 h^{\frac{3}{2}} l^{\frac{1}{2}}$ = the efficiency that would be applied if no more fluid issued than what is due to the actual head (h).

The following Table exhibits the value of both these functions, taking (y) from *o* (where the duty must obviously be nothing) by steps of $\frac{5}{100}$ths to 1.25; after which the duty becomes negative.

y	The Duty for a given expenditure of fluid, on the function $\sqrt{1+y^2}\times y - y^3$	y	The Duty performed by the actual expenditure of fluid, or the function $y+y^3-\sqrt{1+y^2}\times y^3$
.05	.04994	.05	.05000
.10	.09950	.10	.10000
.15	.14830	.15	.14997
.20	.19596	.20	.19984
.25	.24207	.25	.24952
.30	.28621	.30	.29881
.35	.32794	.35	.34762
.40	.36681	.40	.38607
.45	.40234	.45	.41159
.50	.43402	.50	.48525
.55	.46132	.55	.52649
.60	.48371	.60	.56410
.65	.50062	.65	.59706
.70	.51146	.70	.62432
.75	.51563	.75	.64454
.80	.51250	.80	.65632
.85	.50145	.85	.65812
.90	.48183	.90	.64824
.95	.45297	.95	.62479
1.00	.41421	1.00	.58578
1.05	.36488	1.05	.52908
1.10	.30427	1.10	.45233
1.15	.23170	1.15	.35311
1,20	.14646	1.20	.22878
1.25	.04785	1.25	.07660
1.28	—.01803 negative	1.28	—.02929 negative

$l = 16.0954$ feet the log. 1.2067016

If the angular velocity of a body in one second of time be expressed in whole revolutions in degrees, in minutes, or in seconds, the absolute velocity will be =

$$R \times 2cr = Rr \times 6.2831853 \quad \ldots \quad \text{log. } 0.7981799$$

$$D^{\circ} \times \frac{2cr}{360} = D^{\circ} r \times 0.0174533 \quad \ldots \quad \text{log. } 8.2418774$$

$$M' \times \frac{2cr}{360 \times 60} = M' r \times 0.00029089 \quad . \text{ log. } 6.4637261$$

$$S'' \times \frac{2cr}{360 \times 60 \times 60} = S'' r \times 0.000004848817 \text{ log. } 4.6855748$$

The first of these functions reaches the maximum, when y exceeds .75 or $\frac{3}{4}$th by two or three thousandths; and the second when y is about .83.

From an inspection of the Table, it appears that when the machine is moving with the velocity productive of its greatest relative effect, namely, at three quarter parts of the velocity with which the fluid would issue under a pressure equal to that of the actual head, that the duty exceeds by a mere trifle one half of the efficiency expended; and from thence it obviously follows, that the recoil engine cannot in any case be employed with advantage. The water wheel, whenever the fall does not exceed forty or fifty feet; and the pressure engine, provided with an air vessel, in all cases where the fall is greater, receive an efficiency almost equal to the whole expended, instead of one half; and in practice, the duty will reach three quarters. The wheel moreover possesses a self-regulating power, sufficiently accurate for the present illustration, and somewhat analogous to the isochronism of the pendulum; that is, the actual weight of water on the wheel, and its velocity of rotation, must always be reciprocal to each other; thus maintaining a given efficiency, independent of velocity. The received efficiency of a water wheel being represented by the pounds of water passing over it × by the fall in feet, less the height due to the velocity with which the periphery moves in its rotation. But in the recoil engine, the moving power is expending at the rate due to the height even when the machine is actually standing still; and when it moves with a velocity exceeding that due to the length of the head, in the proportion of about 128 : 1 (where the duty is again nothing) the moving power will be expended at the increased rate of $\sqrt{1 + (1.28)^2} : 1$.

Here too may be easily shown the utter impossibility of executing a plan, which would not indeed deserve any notice, were it not that two ingenious practical engineers have within these few years, although at different times, and independently of each other, incurred large expense, and employed much pains in attempting to apply steam on the principle of recoil.

It is obvious that one of the factors, namely force (or the pressure of recoil), must in this case be extremely small, since an aperture of one inch square will discharge about $3\frac{2}{10}$ imperial gallons of water each minute in the form of steam, unassisted by centrifugal force, while the pressure amounts to no more than 14,66 pounds. The other factor (velocity) must therefore be made proportionably great to produce an adequate effect.

Now, since air rushes into a vacuum with the velocity of 1295 feet in a second - - - log. 3.1122698

And steam at 212° is said to have a specific gravity compared to air of 0.6235

Log. 9.7948365 ÷ 2 = 9.8974183 arith. comp. 0.1025817

A velocity of 1640 feet in a second 3.2148515

Three quarters of this velocity in rotation will be requisite for imparting an efficiency to the machine of about one half of the efficiency expended. But such a velocity nearly equals that of a cannon shot, and the effect of centrifugal force on the arms $\left(\frac{x^2}{4l}\right)$ would produce a strain equal to the weight of 41776 feet of the material composing them, independently of the strain arising from centrifugal force of the fluid they contain. And as the modulus of tenacity for iron is estimated at 14800 feet, the centrifugal force will exceed that tenacity almost three times. But independently of this insuperable obstacle, any velocity adequate to the production of an useful efficiency must be utterly unmanageable in every other respect; nor is it easy to imagine the possibility of any expedient, similar to the air pump of Mr. WATT, for maintaining the vacuum in which their arms are to revolve.

The Steam Engine.

To estimate the efficiency of steam acting uniformly with its entire force, at a power corresponding to 30 inches of barometrical pressure, in reference to the consumption of one bushel of coal, I assume, from repeated experiments, that

14 cubic feet of water may be converted into steam, by that given consumption of fuel, occupying about 1330 times more space than the water from which it was produced. The whole space therefore filled by the steam amounts to 18620 cubic feet, capable of containing 1'150410 pounds of water; and as 30 inches of mercury at a specific gravity of 13.568 correspond to a column of water 33.92 feet in length, through this space the 1'150410 pounds of water may be supposed to be raised when a vacuum is formed, giving an efficiency of 39'361000, or above thirty nine millions of pounds raised one foot high.

But when this power of steam is applied through the medium of a piston, acting in the cylinder of a steam engine, large deductions must be made to arrive at the practical duty.

The air pump is usually constructed of a size to require about one eighth part of the efficiency;

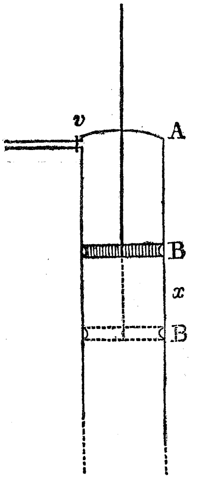
Some imperfection in the vacuum cannot be avoided;

And friction must exist to a considerable amount where bodies in motion have to confine an elastic vapour, ready to escape through the smallest aperture. So that thirty millions would probably have exceeded the attainable limit of duty, but for two expedients:

First. Causing the steam to act expansively, after exerting its whole force through a certain part of the cylinder; and

Secondly. Raising the temperature much above 212° of Fahrenheit, and thereby communicating to the steam a power beyond what is termed atmospheric strength; but this is, of course, accompanied by an additional expenditure of fuel.

The plan of working expansively is founded on the following investigation.



Let A B represent the portion of the cylinder through which the piston is urged by the entire force of the steam, and let its length be expressed by unity. Then let the supply of steam from the boiler be cut off by the valve at (v) and conceive the piston to have descended to B through the space (x) urged by the expansive elasticity of the steam, the density of which will then be $\frac{1}{1+x}$, and assuming that densities and elasticities are proportionate, $\frac{\dot{x}}{1+x}$ will be the fluxion of the efficiency, and the efficiency itself equal the nat. log. of $\overline{1+x}$. And the efficiency of the whole stroke from A to B will be $1 +$ nat. log. $\overline{1+x}$, giving this very curious result, that when x is infinite, the efficiency also exceeds any assignable limit.

$x=1$	$1 + n.l.\overline{1+x} = 1.693$
2	2.099
3	2.386
4	2.609
5	2.792
6	2.946
7	3.079
8	3.197
9	3.303
10	3.398

But it is obvious that several important circumstances have been omitted:

The reduction of temperature, and consequently of elasticity, caused by rarefaction, as the steam expands:

The increased relative effect occasioned by the reaction of the imperfect vacuum:

The increased relative burden of the air pump.

The relative increase of friction.

So that although this mode of working is highly advantageous, yet it is probable that, in practice, not more than from one half to two thirds of the cylinder can beneficially be occupied by expansive steam.

This mode of working has yet a further beneficial effect. The steam, beginning to act with its entire force, imparts the requisite velocity, by rapidly overcoming the inertia; and this force diminishing in the latter part of the stroke, allows the machine to exhaust the motion imparted, as it tends towards its close; thus also avoiding all violent percussion, which must obviously prove most injurious to the machinery, and in other respects be a pure loss.

It would be unfair towards the memory of a very ingenious man, were I to omit noticing, that much about the time when Mr. WATT introduced this method of using the expansive power of steam, a modification of the same principle was invented by Mr. JONATHAN HORNBLOWER, of Penryn in Cornwall. According to this method, instead of condensing the steam after it had passed under the piston, in the working cylinder, the steam is allowed to expand itself over a second piston in a second cylinder, when it is finally condensed in the usual manner; and if the second cylinder exceeds the first in the proportion of $1 + x$ to 1, the theoretical effect will be identical with that produced by the former method. But the mode invented by Mr. WATT excels the other in simplicity, and causes less friction.

The second expedient, that of employing steam at an high degree of temperature, and consequently of pressure, suggested itself in the following manner.

Experiments had shown, that a quantity of heat about nine hundred times greater than what is required to raise the temperature of a given weight of water one degree of Fahrenheit's thermometer, becomes latent in the conversion of that water into steam. That an elevation of temperature to

40° of Fahrenheit above 212° doubles the steam's elastic force, and that 30° more treble it; great expectations were consequently formed of the increased power that would be obtained, from a given consumption of fuel, by using high pressure steam; but the following most important circumstances were overlooked, till experience forced them into view.

That the temperature of steam cannot be raised, with a speed suited to mechanical purposes, in any other way than through the medium of its generating water.

And that water heated above 212° sends off more steam, with a proportionate absorption of latent heat, increasing the density of the steam previously generated of atmospheric elasticity, thus doubling its power at 212° + 40°; but at an expense of fuel, not much inferior to the power gained.

If steam could be heated, as an independent elastic fluid, expanding $\frac{1}{480}$th of its bulk for each degree of Fah.; and if its capacity were the same as the capacity of water, a quantity of heat, equal to that rendered latent, would raise the power from unity to $\left.\frac{481}{480}\right)^{900} = 6\frac{1}{2}$. If the capacities of steam and water, according to some recent experiments, be as 1,55 to 1, the increase of power will be from unity to $\left.\frac{481}{480}\right)^{\frac{900}{1,55}} = 3\frac{1}{3}$. But the non-conducting property, common to all elastic fluids, has hitherto prevented this advantage from being obtained. Heated through the medium of its generating water, high pressure steam is believed however to be advantageous; and by its means additional power may be obtained, when an increase is requisite for any temporary use.

A contrivance has recently been attempted, by which a given small quantity of water is forced into a minute boiler at each stroke of the engine, and the boiler, presenting a very large surface in proportion to its content, is kept at an equable high temperature by immersion in a fuzed metal. Thus great degrees of elasticity and of rarity may be combined;

and the plan seems worthy of being fully prosecuted. But the main hope of augmented power, for still further elevating the condition of the human race, by increasing the empire of intellect over brute force, and for accelerating that splendid career, which, originating with the inventions of the plough and of the loom, has acquired such unhoped for rapidity in our own times, appears to rest on the application to mechanical purposes of some fluid more elastic than the vapour of water, according to the suggestion of our President in the Philosophical Transactions for 1823.

The following are a few instances selected from one of the Monthly Statements, published in Cornwall, of the work done by the principal steam engines, July 1826.

Mines.	Engine, and the diameter of the cylinder.	Load per sq. inch, on the piston.	Length of the stroke, in the cylinder	No. of Lifts	Depth.	Diameter of the Pump.	Time.	Consumption of coal, in bushels.	Number of strokes.	Length of the stroke, in the pump.	Load, in pounds.	Pounds lifted one foot high by consuming a bushel of of coal.	No. of strokes, per minute.	Remarks, and Engineer's Names.
		Lbs.	Ft. In.		Fms. ft.	Inches.				Ft. In.				
Wheal Vor	Trelawny's engine, 80 inches, single.	13,3	9 9	5 3 1 1 1	135 2 33 3 11 4 12 0 7 0	15 16 16 $9\frac{1}{2}$ 9	July 7th to July 31st.	2633	210490	7 0	89305	49,975,186	6,08	Drawing perpendicularly 135 fms.; and on the underlay 27 f. Main beam over the cylinder. Two balance bobs under ground. SIMS and RICHARDS.
Wheal Hope	60 inches single.	6,78	9 0	2 1 1 1	37 4 22 1 5 0 12 1	13 $12\frac{1}{8}$ 8 7	June 28th to July 29th	945	273730	8 0	21560	49,960,794	6,1	Drawing all the load perpendicularly. Main beam over the cylinder. 1 balance bob at the surface. GROSE.
Herland	Manor, 80 inches, single.	10,9	9 6	1 3 1	35 2 112 4 23 2	12 15 $14\frac{7}{8}$	June 27th to July 29th	2619	208880	7 6	72817	43,556,743	4,5	Drawing all the load perpendicularly. Main beam over the cylinder. One balance bob under ground, and one at the surface. WEBB.
Ditto...	Fancy, 80 inches, single.	7,6	9 6	2 1 1	72 0 30 3 21 0	$14\frac{1}{2}$ $14\frac{1}{4}$ 13	Ditto	1825	213670	7 6	50890	44,686,299	4,6	Drawing all the load perpendicularly. Main beam over the cylinder. One balance bob under ground. WEBB.

From this document it appears, that several of the large engines now at work in Cornwall, are actually performing a duty about one quarter part greater than would be the whole efficiency of steam, unaided by expansive working, or by high pressure; assuming that fourteen cubic feet of water is the quantity converted into steam by a bushel of coal; but other engines, apparently similar in every respect, fail in performing half this duty; and no satisfactory cause has yet been assigned for the important difference.

As examples of the power and energies of the human mind, these applications of fire, through the medium of an elastic fluid, to mechanical purposes, stand pre-eminently distinguished: as elaborate contrivances useful to man, steam engines are confessedly without a rival.

No series of investigations can therefore be more worthy of minute and accurate attention than one which may enable us to connect by general laws, the temperatures, the densities, the elasticities, the capacities for heat, and the quantities rendered latent in assuming the elastic form; first for steam, and then, if possible, for all other bodies capable of existing in a gaseous state, both in contact with their generating fluids, and in complete separation from them.

I have gone more at length into the principles of the steam engine, than the immediate object of this Paper either requires or can warrant; but I trust that the Society will admit the great importance of the subject as an excuse; a subject with which local circumstances have rendered me familiar during the whole of my life.